The Debate on Net Neutrality: A Policy Perspective

Hsing Kenneth Cheng*
Subhajyoti Bandyopadhyay
and
Hong Guo

Department of Decision and Information Sciences
Warrington College of Business Administration
University of Florida
Gainesville, FL 32611-7169
U.S.A.

* Corresponding author. Phone: (352)392-7068; Fax: (352) 392-5438 e-mail: hkcheng@ufl.edu
This research was made possible through a grant from the Public Utility Research Center in the Warrington College of Business Administration of the University of Florida
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Abstract

Whether to legislate to maintain “net neutrality”, the current status quo of prohibiting broadband service providers from charging online websites for preferential access to their residential and commercial customers, has become a subject under fierce debate. We develop a stylized game-theoretic model to address two critical issues of the net neutrality: (1) who are gainers and losers of abandoning net neutrality, and (2) will broadband service providers have greater incentive to expand their capacity without net neutrality.

We find that if the principle of net neutrality is abandoned, the broadband service provider definitely stands to gain from the arrangement, as a result of extracting the preferential access fees from the content providers. The content providers are thus left worse off, mirroring the stances of the two sides in the debate. Depending on parameter values in our framework, consumer surplus either does not change or is higher, and in the latter case, while a majority of consumers are better off, a minority of them is left worse off with larger wait times to access their preferred content. The social welfare increases when compared to the baseline case under net neutrality when one content provider pays for preferential treatment, but remains unchanged when both content providers pay. We also find that the incentive for the broadband service provider to expand under net neutrality is unambiguously higher than under the no net neutrality regime. This goes against the assertion of the broadband service providers that under net neutrality, they have limited incentive to expand.

Keywords: Net Neutrality, Economics of Net Neutrality, Broadband Service Providers, Content Providers, Consumer Welfare
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1. Introduction

“The ‘net neutrality’ debate has reached a fever pitch as Congress mulls legislation that would allow Internet service providers to charge Web sites for preferred delivery of digital content.”


The recent proposal by broadband providers like Verizon, Comcast and AT&T (among others), to charge popular online websites for preferential access to their residential and commercial customers, has brought about widespread attention in the media (Helm 2006; Waldmeir 2006). The proposal goes right into the heart of the debate on the idea of “Net neutrality” – the phrase that was first coined by Columbia Law School professor Tim Wu, and is used to signify the concept that the Internet is merely a carrier of online content which does not distinguish one web site from another. The central idea inherent in this concept is that a “maximally useful public information network aspires to treat all content, sites, and platforms equally” (Wu 2003), and while a formal definition of the operationalization of the principle does not exist, (Hahn and Wallsten 2006) point out that it “usually means that broadband service providers charge consumers only once for Internet access, do not favor one content provider over another, and do not charge content providers for sending information over broadband lines to end users.”

Popular online content providers like Google, Yahoo!, Microsoft and others would like to maintain the current status quo, which they claim would preserve the egalitarian philosophy on which the Internet was founded. Other supporters of the concept include online start-ups, who claim that it would be almost impossible for them to pay these proposed fees when their revenue streams are almost non-existent, since they have to give away most of their content for free in order to build a loyal customer base (Sydell 2006).

Some venture capitalists have even argued that abandoning net neutrality would result in would-be entrepreneurs becoming more hesitant to start a business, a state of affairs that might even hurt the competitiveness of the American online firms in the long run (Sydell 2006; Wu 2006a). Vint Cerf, the
renowned computer scientist who is commonly referred to as one of the “founding fathers of the Internet”, contends that such a payment structure would result in the Internet resembling more and more like the controlled mass media today, where the few Internet service providers (ISPs) control what the customers effectively have access to (Waldmeir 2006). Tim Berners-Lee, the founder of the World Wide Web, also favors keeping net neutrality in place, since “[the internet] is the basis of a fair competitive market economy.” (Berner-Lee 2006). Finally, some people have voiced their fears of the Internet service providers starting to offer services like Internet telephony to their consumers at rates that undercut other rival providers who would struggle to remain competitive if they have to pay these fees. This again might result in stagnation in what has so far remained one of the most open marketplaces.

The Internet service providers, on the other hand, have argued that it is they who have put their resources to maintain and upgrade the physical services that they provide to the consumers, and that the popular web sites have so far got a “free ride” on their resources (Waldmeir 2006), and that the “Internet service providers should be allowed to strike deals to give certain Web sites or services priority in reaching computer users” (Krim 2005). With online content increasing exponentially over the years, and consumers increasingly becoming used to broadband access, it will be necessary to meet the rising costs of increasing the capacity and serving the expanded consumer base. Not having these sources of revenue might act as a disincentive to upgrade their infrastructure and affect the service providers’ plans of increasing their existing capacities. That, in turn, would affect many emerging online services like real-time broadband video that by design require preferential treatment of their packets. In some ways, the ISPs contend, the new payment mechanisms might herald the beginning of new business models that demand preferential treatment of their packets, and that the “vertical integration of new features and services by broadband network operators is an essential part of the innovation strategy companies will need to use to compete and offer customers the services they demand” (Thierer 2004). Finally, the ISPs state that abandoning net neutrality will not degrade the service for its customers. Michael McCurry, the co-chairman of a phone industry lobbying organization that has supported the cause of the ISPs, and is
backed by AT&T and Verizon, states that his clients “want to create products and services that will improve the Internet, not degrade it for a certain set of customers” (WSJ 2006).

The proper usage and context of the term “net neutrality” itself has been subject to confusion (Wu 2006b), and an extensive discussion of the issues might be found in (Wu 2003). In brief, network neutrality aims to address concerns raised by some specific behavior of the broadband service providers: (a) blocking of some content providers; (b) preferential treatment of one provider over another and (c) transparency failures, whereby a broadband provider fails to notify its customers and content providers what service they offer in terms of estimated bandwidth, latency, etc. (Wu 2006b). The current proposals by the broadband service providers has raised concerns around the second issue – i.e., the possibility that one content or application provider pays the broadband service provider for preferential treatment of its packets, as the latter acts effectively as a gatekeeper between the content providers and the customers it serves.

The entire debate has raised a number of unanswered questions that are of interest to researchers and practitioners alike, not to mention the regulatory agencies. The intensity of the public debate, and the stakes involved in the issue, was brought into focus during a House Committee hearing in April 2006 (Wu 2006a), where it was pointed out that “[the internet] has become part of America’s basic infrastructure. It has become as essential to people and to the economy as the roads, the electric grid, or the telephone…Given this infrastructure, Americans are accustomed to basic rights to use the network as they see fit.” The discussion is now slated to be part of the agenda of the US Senate in 2007 (Dunbar 2006).

From a policy perspective, two issues are of particular interest. First, the regulatory agencies would like to know who are the gainers and losers if the principle of net neutrality is abandoned. Specifically, if the net social welfare increases as a result of abandoning net neutrality – and more specifically, the end consumers are better off, the idea for the proposed payment mechanisms would gain

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1 For the purposes of this exercise, we use the terms content and applications interchangeably throughout the paper; similarly, the terms broadband service provider and internet service provider (or ISP) have been used interchangeably throughout.
traction among the policymakers; conversely, if abandoning the principle of net neutrality results in helping to extract more rent for only a few private agencies, the idea would find a much less sympathetic audience. In the first part of our paper, we stylize the net neutrality issue in a game-theoretic setup, where we find out the equilibrium in the strategies employed by the content providers, which in turn prompts the appropriate profit maximization strategy of the internet service provider.

The second issue of interest to policymakers is to check the veracity of a key claim of the ISPs – that under net neutrality, the incentive to expand the capacity and the capabilities of the existing infrastructure for the next generation of broadband services is much lesser as compared to when they are allowed to charge the online content providers for preferential treatment. For policymakers, this is indeed a key issue. Higher capacity broadband services will enable many services that are deemed important for the society as a whole. Some examples of such services include disaster recovery, remote medical supervision and the like. For content providers, the next generation broadband services will enable instant delivery of high-definition movies, consumer interactivity, a richer online shopping experience and so forth, and in the process open many new channels of revenue generation. In fact, many consumers today who do not feel the need for broadband services for their typical internet activities like email or online shopping might be ready to pay for such broadband services (Bandyopadhyay and Cheng 2006). In the second half of our paper, we model this long-term scenario (effectively treating capacity as a strategic variable for the internet service provider in the long run, as opposed to it being a constant in the short-run problem) and find out if indeed the internet service providers have a greater incentive to expand their capacities if the principle of net neutrality is abandoned.

From the analytical perspective, what drives the problem is the increased latency of the applications and content of those providers who are given less favorable treatment by the ISP – in other words, the packets from these providers face increased congestion, which translates as a delay “disutility” for the end consumers. Thus, a proper analysis of the problem demands the modeling of the objectives of all the three players – the content providers, the consumers and the ISP in between the two aforementioned players (Hahn and Wallsten 2006). This exercise is distinct and different from the two-
player models that analyze the broadband service provider who provides different classes of service to the end consumers (Bhargava and Sun 2005; Bandyopadhyay and Cheng 2006). It is imperative that this distinction is highlighted, since the network neutrality principle has sometimes been misinterpreted as the barrier to the ability of the broadband service provider to charge consumers different prices for different classes of service. As existing broadband service offerings indicate, ISPs today already charge different prices for different classes of service\(^2\), and such price discrimination strategies enhance the social welfare (Edell and Varaiya 1999; Bandyopadhyay and Cheng 2006; Hermalin and Katz 2006). Finally, we are not concerned about how hosting service providers charge content providers for hosting or transmitting their content over the World Wide Web (see Figure 1), or the various ways by which content providers have their content delivered faster to consumers. In the network neutrality debate, the issue is whether the broadband service provider should be allowed to charge the content providers for transmitting their content from its local switching office\(^3\) to the consumers’ residences or offices (in other words, the issue of interest is only at the local loop, the part encircled in dashed line in Figure 1). Thus, the debate is not about how Tier-1 or Tier-2 ISPs charge content providers, but how local Tier-3 ISPs serving the end consumers propose to charge the content providers (for a very good discussion on the three tiers of ISPs, one is referred to Section 1.5 of (Kurose and Ross 2003)).

--- Insert Figure 1 about here ---

Thus, the role of the broadband service provider that we need to model is not as a *producer* of the service of providing hosting services to the content providers (and in most cases, the hosting service provider is different from the local broadband service provider at the consumers’ end) but as that of a *gatekeeper* who determines how the content from the content producers reach the consumers, *after* it reaches the broadband provider’s local switching office.

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\(^2\) In the authors’ neighborhood (in a medium-sized University town), for example, the cable broadband service provider has broadband download speed offerings of an ‘introductory’ 256kbps, for $25 per month (excluding taxes and regular fees), and ‘regular’ speeds of 4Mbps (for $40) and 9Mbps (for $55).

\(^3\) Technically, the service terminates in a cable modem termination system (CMTS) in the cable modem broadband system or the Digital Subscriber Line Access Multiplexer (DSLAM) in the DSL broadband system, both of which have to be within a few miles of the end consumers. This is also called by the name Internet Exchange Point.
The rest of the paper is arranged as follows. Section 2 outlines the stylized model for our analysis. Section 3 analyzes our first research question – the gainers and losers under net neutrality versus payment mechanism model without net neutrality, while Section 4 considers the issue of the incentive to expand capacity for the ISP. Section 5 concludes, by summarizing the policy implications of our analysis and providing some possible directions for future research. We would like to point out however that the objective of this paper is not to make any policy prescriptions, but rather to provide an objective analysis that clarifies some of the germane issues in this ongoing debate.

2. The model

To analyze the problem at hand, consider a stylized model where there are two online content providers, whom we denote by $G$ and $Y$ (see Figure 1). Let us assume that the consumers’ preferences for content can be represented by a random variable, such that the current status quo would result in every consumer choosing either one of these competitors. The consumers’ net utility from using the services of either $G$ or $Y$ (let us assume for the time being that both of them provide their basic services for free to the end consumers) depends on their individual preferences, the ‘distance’ of their preferred provider (i.e. either $G$ or $Y$) from their ideal, and the cost of delay that is a result of the general congestion in the network between the ISP and the consumer. Currently, the last cost is the same for all consumers, and does not figure into their (i.e. the consumers’) decisions. The content providers can provide their service for ‘free’ to the end users as they generate revenues from advertisers and associated “click-throughs” of the consumers. Note that not all content providers provide their content for free to the consumers, and the revenue streams for a particular content provider can be different from advertisers. In our model, we consider the revenue generation of the content provider as the average revenue generated (from all sources) per packet requested by the end consumer.

In this paper, we analyze the decision process of a monopolist internet service provider that serves the consumers in a specific geographical market. While the monopoly assumption is a
simplification in some geographies, it is to be noted that unlike many other countries, the extent of competition in the local broadband services market is very limited in the United States\(^4\), so much so that in many places, a single broadband service provider is often a de facto monopolist. The situation is aggravated by the high switching cost of long-term service contracts and incompatible broadband technologies between cable and phone companies. Further, many customers are not qualified for digital subscriber’s line (DSL) broadband services from phone companies because they exceed the three miles distance limit from the phone company’s nearest switching office, making the cable operators the de facto monopolistic broadband service providers in several local markets. Thus, in addition to providing the benefit of making the analysis tractable, the assumption closely reflects the reality of local broadband services in the U.S. market.

To understand the impact of abandoning net neutrality, let us now look at the situation where the ISP now starts charging content providers \(G\) and \(Y\) for preferential treatment of their packets, and suppose without loss of generality, that only \(Y\) decides to pay for the service. As a result, any packet from \(Y\) that is received by the internet service provider as a request from one of its customers now gets preferred treatment to the top of the queue (these packets still face the congestion from other similarly preferred packets from the same content provider). We model the congestion in the network after (Mendelson 1985; Bandyopadhyay and Cheng 2006). Packets from \(G\) do not receive any preferential treatment, and at any point in time, are in fact queued after any packet from \(Y\) that might be requested at that point of time. Depending on the number of \(Y\)’s packet in the channel, some consumers who previously preferred \(G\) might now find the congestion of \(G\)’s packets to have enough disutility so that they might now prefer \(Y\)’s service. It is this setup that we analyze in this paper.

Let \(N\) be the total number of end consumers in our model (i.e. the total number of consumers served by the monopolist ISP). Without loss of generality, we normalize this population to 1. Let \(\lambda\) be the Poisson arrival rate of content requested by each consumer, and it is expressed in packets per unit of

\(^4\) See, for example, http://money.cnn.com/2005/06/16/technology/broadband/index.htm
time. Following (Mendelson 1985), we denote $V(\lambda)$ to be the gross value function of this content for each consumer, assumed to be twice differentiable and concave. Let $\mu$ be the capacity that the ISP provides to the consumers, expressed in packets per unit of time. This capacity constraint of the ISP affects the service that it renders to the consumers in a unique fashion – specifically, we can think of the packets requested by the consumers being serviced in an M/M/1 queuing system where consumers have a disutility of waiting for their packets. This delay cost is denoted as $d$ per unit time, and therefore the disutility of the consumers waiting for packets which have an arrival rate $\lambda$ in a system that has a capacity of $\mu$ equals \( \frac{d}{\mu - \lambda} \), i.e., the delay cost parameter $d$ multiplied by the expected time in such a queuing system $\frac{1}{\mu - \lambda}$. The ISP charges a fixed fee $F$ per unit time to the consumers for its services. If it is allowed to charge the online content providers, the ISP would charge a content provider a price $p$ for the priority transmission of its packets; this is a charge per packet per unit of time. Let $r_Y$ and $r_G$ denote the revenue rate of content provider $Y$ and $G$ respectively per packet for content – in other words, these two variables denote the average rate at which the requests for content from the consumers provide revenues to the content provider from myriad types of advertisers who want to reach these consumers. Finally, $t$ is the “fit cost” per unit of distance from an end consumer’s ideal content in the Hotelling framework (Hotelling 1929) (see Figure 2), and $x$ denotes the marginal consumer indifferent to the content between $Y$ and $G$. For ready reference, a list of the notations we use is provided in Appendix A.

--- Insert Figure 2 about here ---

We assume that customers are homogeneous in terms of having the same rate of requests for content, valuation of content, and delay cost. Further, we assume that customers are uniformly distributed in $[0, 1]$ in terms of their ideal content provider, where content provider $Y$ is located at zero, while content provider $G$ is located at the opposite end of the interval (see Figure 2). Since we are modeling only broadband consumers, we assume that the internet service provider captures all end consumers in $[0, 1]$ under net neutrality (i.e. currently). Further, the ISPs have stated that their intention is not to degrade the
online experience for any current broadband subscriber even if net neutrality is abandoned, and therefore we assume that the internet service provider continues to serve all the current consumers when they start charging content providers for preferential delivery of their packets. In other words, we assume that the consumers’ value function $V(\lambda)$ is high enough so that the utility for the indifferent consumer in any of the scenarios that follow is nonnegative. Without loss of generality, we assume that $r_G > r_Y$, which means that one content provider ($G$) is better than the other in getting the “right” consumers for its advertisers (and its other revenue sources) and therefore can charge higher fees from the latter. We note in advance that this assumption does not affect our research findings.

3. Analysis – gainers and losers

We analyze the model first under net neutrality (hereafter shortened to NN), and then when net neutrality is abandoned in favor of the regime where the ISP can charge the content providers (i.e. no net neutrality, or NNN for short).

**Status quo – under net neutrality (NN)**

The marginal consumer $x_{NN}$ indifferent between content provider $Y$ and content provider $G$ under NN is specified by

$$V(\lambda) - t x_{NN} - \frac{d}{\mu - \lambda} - F_{NN} = V(\lambda) - t (1 - x_{NN}) - \frac{d}{\mu - \lambda} - F_{NN}$$

This leads to $x_{NN} = \frac{1}{2}$.

We analyze the internet service provider’s short-run profit maximization problem under NN. Making a simplifying assumption that the ISP has negligible “running costs”, we get the expression for its profit $\Pi_{NN}$:

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5 The subscript $NN$ refers to the calculations under the assumptions of net neutrality.
\[
\max_\lambda \Pi_{NN} = F_{NN} \\
\text{s.t. } V(\lambda) - tx - \frac{d}{\mu - \lambda} F_{NN} \geq 0, \quad 0 \leq x \leq x_{NN} 
\]
\[
V(\lambda) - t(1 - x) - \frac{d}{\mu - \lambda} F_{NN} \geq 0, \quad x_{NN} \leq x \leq 1
\]  

Note that for \( 0 \leq x \leq x_{NN} \) and \( x_{NN} = \frac{1}{2} \),
\[
V(\lambda) - tx - \frac{d}{\mu - \lambda} F_{NN} \geq V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} F_{NN}.
\]
Likewise, for \( x_{NN} \leq x \leq 1 \) and \( x_{NN} = \frac{1}{2} \),
\[
V(\lambda) - t(1 - x) - \frac{d}{\mu - \lambda} F_{NN} \geq V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} F_{NN}.
\]

The ISP’s short-run problem under NN can thus be simplified as follows:
\[
\max_\lambda \Pi_{NN} = F_{NN} \\
\text{s.t. } V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} F_{NN} \geq 0
\]  

This leads to \( \Pi_{NN}^* = F_{NN}^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} \).

The payoff to content provider \( Y \) is \( x_{NN} \lambda r_Y = \frac{1}{2} \lambda r_Y \), and the payoff to content provider \( G \) is
\[
(1 - x_{NN}) \lambda r_G = \frac{1}{2} \lambda r_G.
\]

**Analysis of pricing strategies when there is No Net Neutrality (NNN)**

Let us now analyze the model when the ISP is allowed to charge the content providers for preferential treatment of the latter’s packets. Without net neutrality (or NNN), the ISP charges a content provider \( p \) per packet per unit of time for priority over its competitor’s packets should its competitor choose not to pay. When both content providers pay the price \( p \), both content providers’ packets receive equal treatment. We outline the sequence of events as follows:
Stage 1: The ISP (for example, Comcast) announces the fixed fee $F$ for end consumers, and the priority pricing of $p$ per packet, for the two content providers.

Stage 2: Based upon the price information in stage 1, the two content providers decide simultaneously whether to pay the priority pricing $p$ for their content delivered to end consumers.

Stage 3: After ISP and content providers make their respective decisions, consumers choose either content provider $Y$ or content provider $G$.

--- Insert Figure 3 about here ---

In order to determine his pricing strategy, the ISP solves the game backwards. We show this in steps 1 and 2 below. In step 3, we consider the resulting surpluses for the various players. These results can then be used by the policymaker who has to decide whether to allow the ISP to charge for preferential service, or continue with net neutrality (i.e. decide on NN or NNN).

Step 1: Given certain $F$ and $p$, determine the best responses for content provider $Y$ and content provider $G$. This equilibrium (neither paying, one of the content providers paying, or both content providers paying) will depend on the specific values of the variables involved.

Step 2: Expecting content providers $Y$ and $G$’s best response (that depends on the specific values of the variables involved), determine the optimal decision $(F^*, p^*)$ for the internet service provider. This will involve a comparison of the profits under the various equilibria arrived at in Step 1. Since the ISP is a monopolist, he will be able to choose the menu of prices that he charges the consumers and the content providers to arrive at the highest profit.

Step 3: For the policymaker, compare the equilibrium under NNN and NN. Specifically, we will need to evaluate the payoff for the ISP and content providers, the consumer surplus, and the social welfare.

**Step 1: Given certain $F$ and $p$, what are the best responses for content provider $Y$ and content provider $G$?**
There are essentially four possible scenarios under NNN: both content providers do not pay; one content provider pays and the other does not (which result in two different scenarios); and both content providers pay.

**Scenario 1**: Both content providers opt for not paying the priority price $p$. This amounts to the same result as in net neutrality (NN).

The indifferent consumer $x_1$ is given by

\[
V(\lambda) - tx_1 - \frac{d}{\mu - \lambda} - F_1 = V(\lambda) - t(1 - x_1) - \frac{d}{\mu - \lambda} - F_1 \text{ which leads to } x_1 = \frac{1}{2}.
\]

The broadband provider then solves his profit maximization problem:

\[
\max_{r_i, p_i} \Pi_1 = F_1 \\
\text{s.t. } V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} - F_1 \geq 0 \quad (4-i) \\
\frac{1}{2} \lambda r_y - (x_y \lambda r_y - x_y \lambda p_y) \geq 0 \quad (4-ii) \\
\frac{1}{2} \lambda r_g - [(1 - x_y) \lambda r_g - (1 - x_g) \lambda p_g] \geq 0 \quad (4-iii)
\]

Here, $x_2$ and $x_3$ are the indifferent consumer in scenario 2 (where $Y$ pays and $G$ does not pay the priority access charges to the ISP) and 3 (where $G$ pays and $Y$ does not pay) respectively. The need for these two variables will become clear in the succeeding paragraphs.

Constraint (4-i) is the participation constraint of the consumer. The rationale for constraint (4-ii) is as follows: Given content provider $G$ does not pay, $\frac{1}{2} \lambda r_y$ is content provider $Y$’s payoff of not paying and $x_y \lambda r_y - x_y \lambda p_y$ is content provider $Y$’s payoff for paying the priority access fees (see scenario 2 – which considers this possibility – below). Constraint (4-ii) thus essentially ensures content provider $Y$’s payoff of not paying the ISP is higher than his payoff when he pays. The existence of constraint (4-iii) is

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6 To facilitate understanding, we have made the *numerical* subscripts (1 through 4) of the various parameters correspond to the different scenarios 1 through 4.
similarly explained: given that content provider \( Y \) is not paying, content provider \( G \)'s payoff is either 
\[
\frac{1}{2} \lambda r_G \text{ when he does not pay the ISP or (1-} x_3 \text{)} \lambda r_G - (1- x_3) \lambda p_1 \text{ when he is paying the ISP (this is described in scenario 3). Constraint (4-iii) thus ensures that content provider } G \text{'s payoff of not paying is higher than the payoff of him paying, and thereby ensuring that content provider } G \text{ chooses not to pay.}
\]

Constraint (4-ii) is equivalent to 
\[
p_1 \geq \frac{x_2 - \frac{1}{2}}{x_2} r_Y .
\]
While Constraint (4-iii) is equivalent to 
\[
p_1 \geq \frac{1}{2 - x_3} r_G .
\]

Since \[x_2 - \frac{1}{2} = \frac{1}{2} - x_3\] and \[r_G > r_Y\], \[p_1 \geq \frac{1}{2 - x_3} r_G .
\]

Therefore the results of the profit maximization problem of the ISP in scenario 1 are:

\[
\Pi_1^* = F_1^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} \text{ and charge the content providers a fee } p_1^* \text{ such that } p_1^* \geq \frac{1}{2 - x_3} r_G .
\]

The best response for both content provider \( Y \) and content provider \( G \) is to Not Pay.

**Scenario 2:** Content provider \( Y \) pays \( p_2 \) while content provider \( G \) chooses not to pay.

In Scenario 2, content provider \( Y \)'s packets are prioritized (and therefore face congestion only to the extent of the traffic from \( Y \)), but content provider \( G \)'s packets are not (whose congestion therefore is a function of the entire traffic). The marginal consumer \( x_2 \), who is indifferent between content provider \( Y \) and content provider \( G \) under NNN, is specified by

\[
V(\lambda) - tx_2 - \frac{d}{\mu - x_2 \lambda} - F_2 = V(\lambda) - t(1- x_3) - \frac{d}{\mu - \lambda} - F_2
\]

This leads to \[x_2 > \frac{1}{2} .\]

The Broadband Provider’s short-run problem under no net neutrality (NNN) is thus:

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7 This equality becomes apparent later in Scenario 3.
\[ \max_{\Pi_2} \Pi_2 = F_2 + x_2 \lambda p_2 \]

\[ s.t. \quad V(\lambda) - tx_2 - \frac{d}{\mu - x_2 \lambda} - F_2 \geq 0, \]

\[ V(\lambda) - t(1 - x_2) - \frac{d}{\mu - \lambda} - F_2 \geq 0, \quad (6) \]

\[ (x_2 \lambda r_y - x_2 \lambda p_2) - \frac{1}{2} \lambda r_y \geq 0, \]

\[ (1 - x_2) \lambda r_G - \left( \frac{1}{2} \lambda r_G - \frac{1}{2} \lambda p_2 \right) \geq 0 \]

The first two constraints are the participation constraints of the consumers who prefer \( G \) and the consumers who prefer \( Y \) respectively. The last two constraints ensure that while content provider \( Y \) will pay, content provider \( G \) will not.

Substituting (5) into (6), one has

\[ \max_{\Pi_2} \Pi_2 = F_2 + x_2 \lambda p_2 \]

\[ s.t. \quad V(\lambda) - tx_2 - \frac{d}{\mu - x_2 \lambda} - F_2 \geq 0, \quad (7-i) \]

\[ p_2 \leq \frac{x_2 - \frac{1}{2}}{r_y}, \quad (7-ii) \]

\[ p_2 \geq \frac{x_2 - \frac{1}{2}}{r_G}, \quad (7-iii) \]

We will now proceed to show that Scenario 2 is indeed not feasible. We compare the right hand side (RHS) of constraints (7-ii) and (7-iii):

\[ \frac{RHS_{(7-ii)}}{RHS_{(7-iii)}} = \frac{r_y}{2x_2 r_G} < 1 \text{ since } x_2 > \frac{1}{2} \text{ and } r_G > r_y. \]
Therefore there is no feasible \( p_2 \) and scenario 2 is not possible. Note that the infeasibility of scenario 2 is driven by the fact (or more correctly, the assumption) that \( r_G > r_Y \).\(^8\) In other words, if \( r_G > r_Y \), we can never have a scenario where content provider \( Y \) decides to pay, and \( G \) does not.

**Scenario 3:** This case is the opposite of Scenario 2. Content provider \( G \) decides to pay the preferential packet treatment price of \( p_3 \) per packet, while content provider \( Y \) chooses not to pay.

We carry out a similar analysis as before. The marginal consumer \( x_3 \) indifferent between content provider \( Y \) and content provider \( G \) under NNN in Scenario 3 is specified by

\[
V(\lambda) - tx_3 - \frac{d}{\mu - \lambda} - F_3 = V(\lambda) - t(1-x_3) - \frac{d}{\mu - (1-x_3)\lambda} = F_3
\]

(8)

It follows that \( x_3 < \frac{1}{2} \).

The ISP’s short-run problem under NNN is given by:

\[
\max_{F_3, p_3} \Pi_3 = F_3 + (1-x_3)\lambda p_3
\]

s.t.

\[
V(\lambda) - tx_3 - \frac{d}{\mu - \lambda} - F_3 \geq 0,
\]

\[
V(\lambda) - t(1-x_3) - \frac{d}{\mu - (1-x_3)\lambda} - F_3 \geq 0,
\]

(9)

\[
x_3\lambda r_Y - \left( \frac{1}{2}\lambda r_Y - \frac{1}{2}\lambda p_3 \right) \geq 0,
\]

\[
[(1-x_3)\lambda r_G - (1-x_3)\lambda p_3] - \frac{1}{2}\lambda r_G \geq 0
\]

The first two constraints are the consumers’ participation constraints, while the last two constraints ensure that this scenario actually holds – i.e. \( Y \) does not pay, but \( G \) does.

Substituting (8) into (9), one has

---

\(^8\) The assumption \( r_Y > r_Y \) does not affect the key results of our analyses. If however, the assumption is reversed, then Scenario 3 instead of Scenario 2 becomes infeasible.
\[
\max_{p_3} \Pi_3 = F_3 + (1 - x_3)\lambda p_3
\]

s.t. \[V(\lambda) - tx_3 - \frac{d}{\mu - \lambda} - F_3 \geq 0, \quad (10-i)\]

\[p_3 \geq \frac{1}{1 - x_3} r_Y, \quad (10-ii)\]

\[p_3 \leq \frac{1}{1 - x_3} r_G \quad (10-iii)\]

We compare the RHS of (10-ii) and (10-iii):

\[
\frac{RHS_{(10-ii)}}{RHS_{(10-iii)}} = \frac{2(1 - x_3)r_Y}{r_G}. \quad \text{To analyze the magnitude of this ratio, we consider two cases: Case I:} \]

\[r_G < 2(1 - x_3)r_Y, \quad \text{and Case II:} \quad r_G \geq 2(1 - x_3)r_Y. \]

If Case I is true, there is no feasible \( p_3 \) and scenario 3 is not possible.

If Case II is true, the ISP’s optimal choice of the prices it charges and its profits are given by:

\[F^*_3 = V(\lambda) - tx_3 - \frac{d}{\mu - \lambda}, \quad p^*_3 = \frac{1}{1 - x_3} r_G, \quad \text{and} \]

\[\Pi^*_3 = F^*_3 + (1 - x_3)\lambda p^*_3 = V(\lambda) - tx_3 - \frac{d}{\mu - \lambda} + \left(\frac{1}{2} - x_3\right) \lambda r_G. \]

The payoff to content provider \( Y \) is given by: \( x_3 \lambda r_Y < \frac{1}{2} \lambda r_Y \), while the payoff to content provider \( G \) is given by: \((1 - x_3)\lambda r_G - (1 - x_3)\lambda p^*_3 = \frac{1}{2} \lambda r_G.\)

We note that replacing \( x_2 \) by \((1 - x_3)\) in Equation (5) gives us Equation (8). This implies that \( x_2 = 1 - x_3 \), which in turns implies \( x_2 - \frac{1}{2} = \frac{1}{2} - x_3 \). In other words, the number of additional end
consumers gained by content provider $Y$ in Scenario 2 will be the same as that of content provider $G$ in Scenario 3.

**Scenario 4:** Both content providers pay the priority pricing $p_4$ to have their content delivered. Since both content providers’ packets are treated the same, this leads to $x_4 = x_i = \frac{1}{2}$.

The ISP now solves the following optimization problem.

$$
\begin{align*}
\max_{F_4, p_4} & \quad \Pi_4 = F_4 + \lambda p_4 \\
\text{s.t.} & \quad V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} - F_4 \geq 0, \quad (11-i) \\
& \quad \left(\frac{1}{2} \lambda r_Y - \frac{1}{2} \lambda p_4 \right) - x_3 \lambda r_Y \geq 0, \quad (11-ii) \\
& \quad \left(\frac{1}{2} \lambda r_G - \frac{1}{2} \lambda p_4 \right) - (1 - x_2) \lambda r_G \geq 0 \quad (11-iii)
\end{align*}
$$

From (11-ii), we get
$$
\frac{1}{2} \lambda p_4 \leq \left(\frac{1}{2} - x_3 \right) \lambda r_Y.
$$

From (11-iii), we get
$$
\frac{1}{2} \lambda p_4 \leq \left( x_2 - \frac{1}{2} \right) \lambda r_G.
$$

Since $x_2 - \frac{1}{2} = \frac{1}{2} - x_3$ and by assumption, $r_G > r_Y$, $\left( x_2 - \frac{1}{2} \right) \lambda r_G > \left( \frac{1}{2} - x_3 \right) \lambda r_Y$. Therefore constraints (11-ii) and (11-iii) can be replaced by the constraint: $\frac{1}{2} \lambda p_4 \leq \left( \frac{1}{2} - x_3 \right) \lambda r_Y$. It follows that the ISP’s optimal pricing strategy is given by $F_4^* = F_1^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$, and $p_4^* = (1 - 2x_3) r_Y$, and his profit is $\Pi_4^* = F_4^* + \lambda p_4^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1 - 2x_3) \lambda r_Y$.

The payoff for content provider $Y$ is given by $\frac{1}{2} \lambda r_Y - \frac{1}{2} \lambda p_4^* = x_3 \lambda r_Y$, while the payoff for content provider $G$ is given by $\frac{1}{2} \lambda r_G - \frac{1}{2} \lambda p_4^* = \frac{1}{2} \lambda (r_G - (1 - 2x_3) r_Y)$.
We summarize the results of step 1 in Table 1.

--- Insert Table 1 about here ---

We remind the readers that \( r_G \geq 2(1 - x_3) r_Y \) in Table 1 is the necessary requirement for Scenario 3 to be possible. When this condition is not satisfied, the corresponding Case I result does not exist, and therefore we have only two potential equilibria (of Scenarios 1 and 4) when \( r_G < 2(1 - x_3) r_Y \). Remember further that Scenario 2 is never possible as long as \( r_G > r_Y \).

**Step 2: Expecting content provider Y and G’s response, what is the optimal decision \((F^*, p^*)\) for the ISP?**

We now proceed to analyze the optimal pricing decision of the ISP, armed with the analysis of the previous step. Since the ISP is a monopolist gatekeeper between the content providers and the customers, he can essentially “drive” the direction of the equilibrium of Step 1 in such a way that it ensures the highest possible profits for him.

In Case I, we can readily observe that \( \Pi'_4 > \Pi'_1 \), and therefore the broadband provider will set the final \((F^*, p^*)\) to:

\[
F^* = F'_4 = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} , \quad p^* = p'_4 = (1 - 2x_3) r_Y , \quad \text{to get the profit}
\]

\[
\Pi^* = \Pi'_4 = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1 - 2x_3) \lambda r_Y .
\]

In Case II, we can confirm that \( \Pi'_3 > \Pi'_1 \) (and we have already seen that \( \Pi'_4 > \Pi'_1 \)) since

\[
\Pi'_3 - \Pi'_1 = \left[ V(\lambda) - tx_3 - \frac{d}{\mu - \lambda} \right] - \left[ V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} \right] = \left( \frac{1}{2} - x_3 \right) (t + \lambda r_G) > 0
\]

The ISP thus needs to compare \( \Pi'_3 \) to \( \Pi'_4 \) in order to determine his maximum profit scenario, which leads to:
\[ \Pi_3^* - \Pi_4^* = \left[ V(\lambda) - tx_3 - \frac{d}{\mu - \lambda} + \left(\frac{1}{2} - x_3\right)\lambda r_G \right] - \left[ V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1 - 2x_3)\lambda r_y \right] \]
\[ = \left(\frac{1}{2} - x_3\right)(t + \lambda r_G - 2\lambda r_y) \]

We see that if \( r_G \geq 2r_y - \frac{t}{\lambda} \), then \( \Pi_3^* \geq \Pi_4^* \). The broadband provider will then set the final

\( (F^*, p^*) \) to \( F_3^* = V(\lambda) - tx_3 - \frac{d}{\mu - \lambda}, \quad p^*_3 = p_3^* = \left(\frac{1-x_3}{1-x_3}\right) r_G \), and will get the profit

\[ \Pi^* = \Pi_3^* = V(\lambda) - tx_3 - \frac{d}{\mu - \lambda} + \left(\frac{1}{2} - x_3\right)\lambda r_G. \]

Conversely, if \( r_G < 2r_y - \frac{t}{\lambda} \), then \( \Pi_3^* < \Pi_4^* \). The broadband provider will then set the final

\( (F^*, p^*) \) to \( F_4^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}, \quad p^*_4 = p_4^* = (1 - 2x_3) r_y \), to get a profit of

\[ \Pi^* = \Pi_4^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1 - 2x_3)\lambda r_y. \]

We summarize the results of step 2 in Table 2. Note that the twin conditions of \( r_G \geq 2(1-x_3)r_y \) and \( r_G \geq 2r_y - \frac{t}{\lambda} \) can be combined as \( r_G \geq \max\left\{2(1-x_3)r_y, 2r_y - \frac{t}{\lambda}\right\} \) in Table 2.

Likewise, \( r_G < 2(1-x_3)r_y \), or \( r_G \geq 2(1-x_3)r_y \) and \( r_G < 2r_y - \frac{t}{\lambda} \) can be simplified as

\( r_G < \max\left\{2(1-x_3)r_y, 2r_y - \frac{t}{\lambda}\right\} \).

--- Insert Table 2 about here ---

Figure 4 condenses these results graphically, by plotting \( r_G \) against \( r_y \). In Region A, where \( r_G \) is “significantly” greater than \( r_y \), the ISP chooses his pricing strategy in such a way to drive the game in Step 1 to scenario 3 – i.e., the higher revenue content provider pays and the lower revenue content
provider does not pay for preferential access; conversely, when the content providers have more “comparable” revenue rates (region B), the game will end up with Scenario 4, whereby both content providers will end up paying.

--- Insert Figure 4 about here ---

It is to be noted that the choice of making $r_G > r_Y$ is simply a matter of convenience of exposition, and that the generalized results would be symmetric on either side of the line $r_G = r_Y$. This is indicated in Figure 5. Regions C and D can be analogously interpreted as Regions A and B.

--- Insert Figure 5 about here ---

**Step 3: Compare the equilibrium under NNN to the NN case.**

We finally look at the problem from the viewpoint of the policymaker by accounting for the surpluses of the various players in this analysis. It is clear from the ISP’s point of view that he would prefer NNN over NN (the profits in either Case A or Case B – summarized in Table 2 – is higher than $\Pi_{NN}$). What is of interest to the policymaker is whether the other participants gain by this arrangement too (i.e., whether their surpluses increase or decrease), and whether the social welfare as a whole increases. We summarize the results of this analysis in Table 3, and the reader is urged to look into Appendix B for the details of their derivations.

--- Insert Table 3 about here ---

As is clear from the above results, the gains of abandoning net neutrality are not experienced equally. While the monopolist internet service provider gains from the new settings (both Case A and Case B), the content providers are definitely worse off by the arrangement (only content provider $G$’s surplus is unchanged under Case A). It is therefore no wonder why the content providers and the internet service providers have been on the opposite sides of the net neutrality debate.

The fate of the end consumers is more nuanced. If the two content providers do not differ by a “significant” extent (regions B or D in Figure 5) in terms of their revenue generation rate, the consumer
surplus is unchanged. However, the consumers as a whole do stand to gain if one content provider is significantly better than the other in revenue generation (regions A and C in Figure 5).

Similar conclusions can be drawn about the social welfare. Under NNN Case B, social welfare (like consumer surplus) does not change, but there is a transfer of wealth from the content providers to the internet service provider. This transfer is made possible fully through the priority access fees that the internet service provider extracts from both the content providers, since the subscription fee to the end consumers do not change (since $F_4^* = F_1^*$). Under NNN Case A, the consumer surplus increases even though the consumers pay a higher fixed fee ($F_3^* > F_1^*$). This is made possible by the fact that the consumers of content provider $G$ (who are a majority) enjoy reduced congestion at the expense of the consumers of content provider $Y$, which more than offsets the loss of welfare of the minority “$Y$-consumers” and the effect of the higher subscription fees on all consumers. It is to be noted in passing that this result goes against the assertion of the ISPs that no consumer would be left worse off under the new arrangement. The broadband service provider stands to gain both from the increased subscription fees as well as the rent extracted from content provider $G$. It is interesting to note that $G$ does not get to enjoy the increase in the number of consumers, since that rent is fully extracted by the internet service provider.

4. Does NN hinder the broadband service provider's incentive to expand capacity in infrastructure?

As discussed earlier, the other key question for the policymaker is the internet service provider’s motivation to expand under NN. To discuss this question, we need to consider the cost of capacity expansion (this was not an issue in the preceding analysis, since in the short run the existing capacity is fixed). In Appendix C, we outline some of the current cost structures in the broadband industry. As this empirical data suggests, there are two principal components to this cost. The first is a fixed component, which comprises of the cost of the cable and the costs of laying it (the same cable can be used for
different capacities). The other component is variable, and depends on the capacity – different transceivers are required to transmit at different data rates, and Appendix C outlines the cost of three different types of transceivers that transmit at speeds of 10 Mbps, 100 Mbps and 1 Gbps. To generalize our discussion, we carry out our analysis for three different “capacities”, \( \mu \), 10\( \mu \) and 100\( \mu \). The total “generalized” cost structures for these three different capacities are summarized in Table 4.

Table 4: Representative Capacities and Costs

<table>
<thead>
<tr>
<th>Capacity</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>( H + k\mu )</td>
</tr>
<tr>
<td>10( \mu )</td>
<td>( H + c_1 k\mu )</td>
</tr>
<tr>
<td>100( \mu )</td>
<td>( H + c_2 k\mu )</td>
</tr>
</tbody>
</table>

Here, \( H \) denotes the fixed cost, and the variable cost component is \( k\mu \), \( c_1 k\mu \), or \( c_2 k\mu \) depending on the capacity level\(^9\). Our objective is to find out the optimal capacity decision under NN and NNN, and compare the two meaningfully in order to find the regime under which the incentive for the ISP to expand capacity is higher. We observe that evaluating the incentive to expand for the ISP is distinct from calculating just the magnitude of his profit.\(^{10}\) Finally, we note that the choice of regime (NN or NNN) is not under the control of the ISP, and is a choice that lies with the policymakers (who can calculate the aforementioned incentives).

The process for solving the problem can be summarized as follows: (1) Plug in each capacity from Table 4 into the short-run problem to find the optimal profit for each capacity; and (2) evaluate and compare the increase in optimal profit to the increase in the cost to install that capacity in order to determine whether that capacity expansion is optimal. The analysis is carried out both under NN and NNN.

---

\(^9\) The two multiplicative factors signify that (1) the current variable cost is a function of the capacity, and (2) moving to a higher capacity involves a non-linear increase in cost. For example, Appendix C suggests that moving to 10 times the current capacity will increase the variable cost component by about 100%, and a hundred-fold increase in capacity will increase the variable cost component 11-fold.

\(^{10}\) A simple example illustrates this. The incentive to expand is greater if the ISP’s profit goes up from $1 to $5 (an increase of $4) by expanding, than when the profit goes up from $5 to $8 (an increase of $3).
For example, under NN the increase of profit per unit of time by expanding capacity from $\mu$ to $10\mu$ is 

$$\Pi_{NN,10\mu}^* - \Pi_{NN,\mu}^* = \frac{d}{\mu - \lambda} - \frac{d}{10\mu - \lambda}. \quad (1)$$

If the discount rate is $\delta$, then the broadband provider will expand the capacity from $\mu$ to $10\mu$ if the net present value of the increased cash flows is greater than the extra initial outlay in capital expenditure:

$$\frac{1}{1-\delta} \left( \frac{d}{\mu - \lambda} - \frac{d}{10\mu - \lambda} \right) \geq (c_1-1) k \mu$$

**The long-run capacity expansion problem**

We treat capacity $\hat{\mu}$ as a decision variable where it can only take three values $\mu$, $10\mu$, or $100\mu$ with the cost $H + k\mu$, $H + c_1 k\mu$, or $H + c_2 k\mu$ respectively. We go through a similar process employed to analyze the short-run problem. Once again, there are three steps:

Step 1: Given certain $F$, $p$, and $\mu$, what are the best responses for content provider $Y$ and content provider $G$? The analysis in the step does not differ from that in Section 4, and is therefore not shown below.

Step 2: Expecting content providers $Y$ and $G$’s responses, we evaluate the optimal decision $(F^*, p^*, \mu^*)$ for the broadband provider.

Step 3: Compare the equilibrium under NNN to the NN case. Specifically, we need to calculate the payoff for the broadband provider and the content providers, the consumer surplus, and the social welfare.

**Step 2 in the capacity expansion problem:**

**Scenario 1:** $x_1 = \frac{1}{2}$

The broadband provider solves the following problem:

---

11 We note that the subscripts of the optimal profits reflect the different capacities.
\[
\max_{\tilde{\mu}, \mu} \Pi_{1, \tilde{\mu}} = \frac{1}{1 - \delta} F_1 - C(\tilde{\mu}) \\
\text{s.t. } V(\lambda) - \frac{t}{2} - \frac{d}{\tilde{\mu} - \lambda} - F_1 \geq 0 \\
\frac{1}{2} \lambda r_y - (x_2 \lambda r_y - x_3 \lambda p_1) \geq 0 \\
\frac{1}{2} \lambda r_G - [(1 - x_2) \lambda r_G - (1 - x_3) \lambda p_1] \geq 0
\]

where \( x_2 \) and \( x_3 \) are as defined in scenario 2 and 3.

Note that the objective function represents the net cash flow for a given capacity \( \tilde{\mu} \). We know that the first constraint is binding, i.e. \( F_1^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu^* - \lambda} \). The optimal capacity \( \mu^* \) is derived by substituting three values of \( \tilde{\mu} \) into the objective function and finding the largest long-term net cash flows in below:

\[
\Pi_{1, \mu} = \frac{1}{1 - \delta} \left( V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} \right) - (H + k \mu) \\
\Pi_{1, 10 \mu} = \frac{1}{1 - \delta} \left( V(\lambda) - \frac{t}{2} - \frac{d}{10 \mu - \lambda} \right) - (H + c_1 k \mu) \\
\Pi_{1, 100 \mu} = \frac{1}{1 - \delta} \left( V(\lambda) - \frac{t}{2} - \frac{d}{100 \mu - \lambda} \right) - (H + c_2 k \mu)
\]

Comparing the three, we conclude that the broadband provider will expand the capacity

from \( \mu \) to \( 10 \mu \) if \( \frac{1}{1 - \delta} \left( \frac{d}{\mu - \lambda} - \frac{d}{10 \mu - \lambda} \right) \geq (c_1 - 1) k \mu \)

and from \( 10 \mu \) to \( 100 \mu \) if \( \frac{1}{1 - \delta} \left( \frac{d}{10 \mu - \lambda} - \frac{d}{100 \mu - \lambda} \right) \geq (c_2 - c_1) k \mu \)

**Scenario 2:** It can be proved analogously as discussed earlier in Section 4 that this scenario is not possible.
Scenario 3:

One has \( x_3 < \frac{1}{2} \), which is determined by \( tx_3 + \frac{d}{\mu - \lambda} = t(1-x_3) + \frac{d}{\mu - (1-x_1)\lambda} \).

It can be shown that \( x_3 \) increases in capacity \( \hat{\mu} \) (the proof appears in Appendix D).

As before, \( F_3^* = V(\lambda) - tx_3 - \frac{d}{\mu_3^* - \lambda} \), and \( p_3^* = \frac{1-x_3}{1-x_3} r_G \).

Substituting the various capacities and their corresponding costs, we compute the long-term net cash flows once again:

\[
\Pi_{3,\mu} = \frac{1}{1-\delta} \left( V(\lambda) - tx_{3,\mu} - \frac{d}{\mu - \lambda} + \left( \frac{1}{2} - x_{3,\mu} \right) \lambda r_G \right) - (H + k\mu)
\]

\[
\Pi_{3,10\mu} = \frac{1}{1-\delta} \left( V(\lambda) - tx_{3,10\mu} - \frac{d}{10\mu - \lambda} + \left( \frac{1}{2} - x_{3,10\mu} \right) \lambda r_G \right) - (H + c_1 k\mu)
\]

\[
\Pi_{3,100\mu} = \frac{1}{1-\delta} \left( V(\lambda) - tx_{3,100\mu} - \frac{d}{100\mu - \lambda} + \left( \frac{1}{2} - x_{3,100\mu} \right) \lambda r_G \right) - (H + c_2 k\mu)
\]

As before, the broadband provider will expand the capacity from \( \mu \) to \( 10\mu \) if

\[
\frac{1}{1-\delta} \left( \frac{d}{\mu - \lambda} - \frac{d}{10\mu - \lambda} \right) - (x_{3,10\mu} - x_{3,\mu})(\lambda r_G + t) \geq (c_1 - 1) k\mu , \text{ and}
\]

from \( 10\mu \) to \( 100\mu \) if

\[
\frac{1}{1-\delta} \left( \frac{d}{10\mu - \lambda} - \frac{d}{100\mu - \lambda} \right) - (x_{3,100\mu} - x_{3,10\mu})(\lambda r_G + t) \geq (c_2 - c_1) k\mu
\]

Scenario 4: \( x_4 = \frac{1}{2} \)

\[ F_4^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu_4^* - \lambda} \] and \( p_4^* = (1-2x_4) r_G \).

The corresponding net cash flows are given by
\[
\Pi_{4,\mu} = \frac{1}{1-\delta} \left( V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1-2x_{3,\mu}) \lambda r_{T} \right) - (H + k\mu)
\]
\[
\Pi_{4,10\mu} = \frac{1}{1-\delta} \left( V(\lambda) - \frac{t}{2} - \frac{d}{10\mu - \lambda} + (1-2x_{3,10\mu}) \lambda r_{T} \right) - (H + c_{1}k\mu),
\]
\[
\Pi_{4,100\mu} = \frac{1}{1-\delta} \left( V(\lambda) - \frac{t}{2} - \frac{d}{100\mu - \lambda} + (1-2x_{3,100\mu}) \lambda r_{T} \right) - (H + c_{2}k\mu)
\]

and the broadband service provider will expand capacity

from $\mu$ to $10\mu$ if

\[
\frac{1}{1-\delta} \left( \left( \frac{d}{\mu - \lambda} - \frac{d}{10\mu - \lambda} \right) - 2 \left( x_{3,10\mu} - x_{3,\mu} \right) \lambda r_{T} \right) \geq (c_{1} - 1)k\mu ,
\]

and

from $10\mu$ to $100\mu$ if

\[
\frac{1}{1-\delta} \left( \left( \frac{d}{10\mu - \lambda} - \frac{d}{100\mu - \lambda} \right) - 2 \left( x_{3,100\mu} - x_{3,10\mu} \right) \lambda r_{T} \right) \geq (c_{2} - c_{1})k\mu
\]

**Step 3 in the capacity expansion problem:**

Summarizing the results in Step 2, we see that the broadband provider will expand the capacity.

**Case (i): from $\mu$ to $10\mu$ if**

in Scenario 1:

\[
\frac{1}{1-\delta} \left( \frac{d}{\mu - \lambda} - \frac{d}{10\mu - \lambda} \right) \geq (c_{1} - 1)k\mu
\]

in Scenario 3:

\[
\frac{1}{1-\delta} \left( \left( \frac{d}{\mu - \lambda} - \frac{d}{10\mu - \lambda} \right) - \left( x_{3,10\mu} - x_{3,\mu} \right) \lambda r_{T} + t \right) \geq (c_{1} - 1)k\mu
\]

in Scenario 4:

\[
\frac{1}{1-\delta} \left( \left( \frac{d}{\mu - \lambda} - \frac{d}{10\mu - \lambda} \right) - 2 \left( x_{3,10\mu} - x_{3,\mu} \right) \lambda r_{T} \right) \geq (c_{1} - 1)k\mu
\]

and

**Case (ii): from $10\mu$ to $100\mu$ if**

in Scenario 1:

\[
\frac{1}{1-\delta} \left( \frac{d}{10\mu - \lambda} - \frac{d}{100\mu - \lambda} \right) \geq (c_{2} - c_{1})k\mu
\]

in Scenario 3:

\[
\frac{1}{1-\delta} \left( \left( \frac{d}{10\mu - \lambda} - \frac{d}{100\mu - \lambda} \right) - \left( x_{3,100\mu} - x_{3,10\mu} \right) \lambda r_{T} + t \right) \geq (c_{2} - c_{1})k\mu
\]
and in Scenario 4: 
\[
\frac{1}{1 - \delta} \left( \left( \frac{d}{10\mu - \lambda} - \frac{d}{100\mu - \lambda} \right) - 2 \left( x_{3,10\mu} - x_{3,100\mu} \right) \lambda r_Y \right) \geq (c_2 - c_1) k \mu 
\]

We note that in both cases (i) and (ii) above, scenario 1 corresponds to the NN regime while scenarios 3 and 4 correspond to the NNN regime. We note further that in the above inequalities, the left hand side corresponds to the quantum increase in long-term net cash flow for the ISP in moving from a lower capacity to a higher capacity, while the right hand side is the sunk cost hurdle to be overcome for the capacity expansion to be profitable. Looking at these expressions, we can easily conclude that while the ISP might make higher profits under NNN, his incentive to upgrade capacity is higher under NN (recall the illustration in footnote 10). This is so because the long-term net cash flow (the left-hand side of the condition) for capacity expansion under NN (i.e. Scenario 1) is greater than those of Scenario 3 and 4 (under NNN)\(^\text{12}\), while the sunk cost hurdle (the right-hand side of the condition) remains the same under NN and NNN. The intuition behind the fact that the ISP has more incentive to expand capacity under NN is as follows: In scenario 1 (i.e. under NN), the ISP’s profit comes from only the consumers: 
\[
V(\lambda) = \frac{t}{2} - \frac{d}{\mu - \lambda}. 
\]
In Scenario 3, the ISP’s profit consists of two parts: first, the fees from the consumers, 
\[
V(\lambda) = t x_3 - \frac{d}{\mu - \lambda} \quad \text{and second, the fees from content provider } G, \quad \left( \frac{1}{2} - x_3 \right) \lambda r_G. 
\]
In Scenario 4, the ISP’s profit consists of three parts: fees from the consumers, 
\[
V(\lambda) = \frac{t}{2} - \frac{d}{\mu - \lambda} \quad \text{; fees from content provider } Y, \quad \left( \frac{1}{2} - x_3 \right) \lambda r_Y \quad \text{and finally, fees from content provider } G, \quad \left( \frac{1}{2} - x_3 \right) \lambda r_G. 
\]
We first compare Scenario 3 to Scenario 1. When capacity increases in Scenario 3, consumers face less congestion as a whole, and therefore value the preferential treatment of content provider \(G\)’s packets relatively less as compared to the case when capacities were lower (and congestion was higher). As a result, some of \(G\)’s

\(^{12}\) Central to this result is the fact that \(\frac{\partial x_3}{\partial \mu} > 0\), the proof of which we provide in Appendix D.
consumers switch to \( Y \) (\( x_3 \) increases in \( \mu \) - see footnote 12), who does not pay the ISP for this privilege (in Scenario 3, only \( G \) pays the ISP). In other words, the quantum of \( G \)'s \textit{increased} surplus that is available to the ISP to extract diminishes in Scenario 3 as compared to Scenario 1, when the capacity is upgraded. However, this effect does not exist in scenario 1 (under net neutrality), since consumers of both \( G \) and \( Y \) face equal congestion. Similarly, when the ISP upgrades capacity, the \textit{difference} in surplus extracted from the consumers in Scenario 3 is lesser than the difference in the surplus extracted from the consumers in Scenario 1. The sum total effect is that under capacity expansion, the quantum of difference in profit for the ISP is lesser in Scenario 3 than in Scenario 1. An analogous explanation exists when we compare Scenario 4 to Scenario 1. When capacity increases, in Scenario 4 (when both \( G \) and \( Y \) pay the ISP), the ISP can charge both \( Y \) and \( G \) only to the extent of the fee that \( Y \) would pay, which is the payoff increase for \( Y \) (as compared to Scenario 3) by paying for the preferential treatment. However, as we showed earlier, with capacity expansion \( Y \) would gain some consumers from \( G \) and therefore the fee \( Y \) would be willing to pay will decrease. As a result, the difference in surplus that ISP can extract from both \( Y \) and \( G \) in Scenario 4 is lesser than that in Scenario 1. Recall that the \textit{difference} in fees extracted under capacity expansion from the consumers is the same under Scenario 1 and 4. The sum total effect once again is that under capacity expansion, the quantum of difference in profit for the ISP is lesser in Scenario 4 than in Scenario 1. Summarizing the two situations, we can see that the ISP has more incentive to expand capacity under NN than under NNN.\(^\text{13}\)

Though not directly related to the issue in question, we make one interesting observation: as the ISP’s capacity increases, the parameter space for Scenario 3 becomes bigger in the expense of Scenario 4.

\(^\text{13}\) We illustrate our findings with some hypothetical values that are consistent with the results of our analysis – specifically, comparisons of NN versus NNN (Table 3) and the comparative statics computations (Table 5). Say the broadband service provider makes profit of $1 under net neutrality with current capacity and this goes up to $5 with expanded capacity. All these profits come from one source – the consumers. Under NNN, the $1 of profit goes up to $3 with current capacity: $1 coming from the consumers, $2 from the content providers. When the capacity is expanded under NNN, the profit goes up to $6: $5 from the consumers (just as under NN), and $1 from the content providers. Thus, the content providers’ inclination to pay actually goes down from $2 to $1 – with higher overall capacity, the advantage for priority handling of one’s packets is lesser. The incentive to expand for the broadband service provider under net neutrality was an increase of $4 ($5 minus $1); under NNN, though the absolute value of the profit ($6) is higher than under net neutrality ($5), the \textit{incentive} to expand is lower at $3 ($6 minus $3).
(the kinked line separating the two scenarios squeezes out Region B in Figure 4). The intuition is straightforward – as capacity increases, congestion goes down and some customers of $G$ prefer $Y$. $Y$ therefore has lesser incentive to pay, which makes Scenario 3 more probable.

For completeness sake, we summarize the comparative statics for the various pricing variables and the surpluses of the various parties involved with respect to the capacity $\mu$ in Table 5.

--- Insert Table 5 about here ---

5. Conclusion - policy implications and future directions

The absence of meaningful competition in providing broadband access to consumers in many areas of the United States makes the broadband service provider a de facto monopolist, and therefore the sole gatekeeper in determining what content gets across to the end consumer, and in what fashion. Therefore the debate about net neutrality assumes tremendous importance to a policymaker. This research aims to answer two issues therein in a stylized framework. We find that if the principle of net neutrality is abandoned, the ISP definitely stands to gain from the arrangement, as a result of extracting the preferential access fees from the content providers. The content providers are thus left worse off, mirroring the stances of the two sides in the debate. Depending on parameter values in our framework, consumer surplus either does not change or is higher, and in the latter case, while a majority of consumers are better off, a minority of them is left worse off with larger wait times to access their preferred content. The social welfare increases when compared to the baseline NN case when one content provider pays for preferential treatment, but remains unchanged when both content providers pay. The crucial parameter that determines the nature of the equilibrium is the relative magnitude of the revenue generation capabilities of the two content providers – if they differ significantly (the extent of the difference is captured graphically in Figures 4 and 5), the consumers of the less “effective” (i.e. in terms of revenue generation) content providers, who are a minority, are left worse off.
We also find that the incentive for the broadband service provider to expand under net neutrality is unambiguously higher than under the no net neutrality regime in the context of our framework. This goes against the assertion of the broadband service providers that under net neutrality, they have limited incentive to expand. The experience in broadband markets around the world support our conclusions. In Japan, for example, fierce competition among broadband service providers has led to the introduction of download bandwidth speeds in excess of 100 Mbps as far back as in 2004 (Yang 2004), with prices for the consumers significantly lower than that in the United States (Turner 2005).

Some immediate areas of future research include the assumption that the broadband providers might decide not to pursue all the current subscribers after all, if that scenario can ensure higher profits. One interesting extension is to study the implication of net neutrality on content providers’ ability to provide premium services (e.g., real-time video or remote medical supervision) that require dedicated bandwidth. Another direction of research would be to consider the effect of the broadband service provider as a potential competitor to the content (or other service) providers. Such a situation already exists today (albeit with limited success so far) with broadband service providers like Comcast building their own modest internet portals, or with providers like AT&T or Comcast offering VoIP digital phone services (Krim 2005). Policymakers would then like to ascertain that the monopolist broadband service provider does not enjoy unfair competitive advantage, and look for guiding principles for ensuring fair competition under NN and NNN (the issue gets more complicated – and interesting – when a service provider like AT&T might end up cannibalizing its own traditional phone service by offering the new product).

In conclusion, this research shines light on two major issues in the ongoing net neutrality debate. The results of our analysis should therefore be of great interest to policymakers as they deliberate on this very important problem.
List of figures and tables

Figure 1: Schematic of the model

World Wide Web

Broadband Provider at local loop

End Consumers

Content Provider #1 (e.g., Company Y)

Content Provider #2 (e.g., Company G)

World Wide Web

Figure 2: The content providers and their share of consumers

Figure 3: The sequence of events in the short-run problem
Figure 4: Graphical representation of the regions for arriving at different equilibria of the game when $r_G > r_Y$

Figure 5: Generalized representation of the regions for arriving at different equilibria of the short-run game
Table 1: Summary of results for Step 1 of the short-run game

<table>
<thead>
<tr>
<th>Condition</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_G &lt; 2(1-x_1)r_\gamma$</td>
<td>There are two potential equilibriums:</td>
</tr>
<tr>
<td></td>
<td>Scenario 1 with $\Pi_1^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$</td>
</tr>
<tr>
<td></td>
<td>Scenario 4 with $\Pi_4^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1-2x_1) \lambda r_\gamma$</td>
</tr>
<tr>
<td>$r_G \geq 2(1-x_1)r_\gamma$</td>
<td>There are three potential equilibriums:</td>
</tr>
<tr>
<td></td>
<td>Scenario 1 with $\Pi_1^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$</td>
</tr>
<tr>
<td></td>
<td>Scenario 3 with $\Pi_3^* = V(\lambda) - tx_3 - \frac{d}{\mu - \lambda} + (\frac{1}{2} - x_3) \lambda r_G$</td>
</tr>
<tr>
<td></td>
<td>Scenario 4 with $\Pi_4^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1-2x_1) \lambda r_\gamma$</td>
</tr>
</tbody>
</table>

Table 2: Summary of results of Step 2 of the short-run game

| Case A: $r_G \geq \max \left\{ 2(1-x_1)r_\gamma, 2r_\gamma - \frac{t}{\lambda} \right\}$ | $F^* = F_3^* = V(\lambda) - tx_3 - \frac{d}{\mu - \lambda}$ |
|                                                                     | $p^* = p_3^* = \left( \frac{1}{2} - x_3 \right) \frac{r_G}{1-x_3}$ |
|                                                                     | $\Pi^* = \Pi_3^* = V(\lambda) - tx_3 - \frac{d}{\mu - \lambda} + (\frac{1}{2} - x_3) \lambda r_G$ |
| Case B: $r_G < \min \left\{ 2(1-x_1)r_\gamma, 2r_\gamma - \frac{t}{\lambda} \right\}$ | $F^* = F_4^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$ |
|                                                                     | $p^* = p_4^* = (1-2x_3) r_\gamma$ |
|                                                                     | $\Pi^* = \Pi_4^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1-2x_3) \lambda r_\gamma$ |
Table 3: Comparison of various parameters of interest under NN and NNN
(The text that is **bold and italicized** shows how those specific parameters change when moving from NN to NNN)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NN (benchmark)</th>
<th>NNN (Case A)</th>
<th>NNN (Case B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>$F_1^* = V(\lambda) - \frac{t}{2} \frac{d}{\mu - \lambda}$</td>
<td>$F_3^* = V(\lambda) - t x_3 - \frac{d}{\mu - \lambda} &gt; F_1^*$</td>
<td>$F_4^* = V(\lambda) - \frac{t}{2} \frac{d}{\mu - \lambda} = F_1^*$</td>
</tr>
<tr>
<td>$p$</td>
<td>N/A</td>
<td>$p_3^* = \frac{1}{2} - x_3$</td>
<td>$p_4^* = (1 - 2 x_3) r_y$</td>
</tr>
<tr>
<td>broadband provider surplus</td>
<td>$\Pi_1^* = F_{NN}^* = V(\lambda) - \frac{t}{2} \frac{d}{\mu - \lambda}$</td>
<td>$\Pi_3^* = V(\lambda) - t x_3 - \frac{d}{\mu - \lambda} + \left(\frac{1}{2} - x_3\right) \lambda r_G &gt; \Pi_1^*$</td>
<td>$\Pi_4^* = V(\lambda) - \frac{t}{2} \frac{d}{\mu - \lambda} + (1 - 2 x_3) \lambda r_y &gt; \Pi_1^*$</td>
</tr>
<tr>
<td>content provider $Y$ surplus</td>
<td>$\frac{1}{2} \lambda r_y$</td>
<td>$x_3 \lambda r_y &lt; \frac{1}{2} \lambda r_y$</td>
<td>$x_3 \lambda r_y &lt; \frac{1}{2} \lambda r_y$</td>
</tr>
<tr>
<td>content provider $G$ surplus</td>
<td>$\frac{1}{2} \lambda r_G$</td>
<td>$\frac{1}{2} \lambda r_G$</td>
<td>$\frac{1}{2} \lambda \left( r_G - (1 - 2 x_3) r_y \right) &lt; \frac{1}{2} \lambda r_G$</td>
</tr>
<tr>
<td>consumer surplus</td>
<td>$\frac{t}{4}$</td>
<td>$t \left( x_3^2 - x_3 + \frac{1}{2} \right) &gt; \frac{t}{4}$</td>
<td>$\frac{t}{4}$</td>
</tr>
<tr>
<td>social welfare</td>
<td>$V(\lambda) - \frac{t}{4} \frac{d}{\mu - \lambda} + \frac{1}{2} \lambda r_y + \frac{1}{2} \lambda r_G$</td>
<td>$V(\lambda) - t \left( x_3^2 - 2 x_3 + \frac{1}{2} \right) - \frac{d}{\mu - \lambda} + x_3 \lambda r_y + (1 - x_3) \lambda r_G$</td>
<td>$V(\lambda) - \frac{t}{4} \frac{d}{\mu - \lambda} + \frac{1}{2} \lambda r_y + \frac{1}{2} \lambda r_G$</td>
</tr>
</tbody>
</table>
Table 5: Comparative statics (with respect to capacity, $\mu$)

(legend: $\boxed{+}$: increasing with $\mu$; $\boxed{-}$: decreasing with $\mu$; $\boxed{?}$: depends on parameter values)

<table>
<thead>
<tr>
<th></th>
<th>NN (benchmark)</th>
<th>NNN (Case A)</th>
<th>NNN (Case B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F^*$</td>
<td>$F_1^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$ $\boxed{+}$</td>
<td>$F^* = F_3^* = V(\lambda) - tx_3 - \frac{d}{\mu - \lambda} &gt; F_1^*$ $\boxed{?}$</td>
<td>$F^* = F_4^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} = F_1^*$ $\boxed{+}$</td>
</tr>
<tr>
<td>$p^*$</td>
<td>N/A</td>
<td>$p^* = p_3^* = \frac{1}{2} - x_3$ $\boxed{-}$</td>
<td>$p^* = p_4^* = (1 - 2x_3)r_y$ $\boxed{-}$</td>
</tr>
<tr>
<td>broadband provider $\Pi^<em>_1 = F_1^</em> = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$ $\boxed{+}$</td>
<td>$V(\lambda) - tx_3 - \frac{d}{\mu - \lambda} + \left(\frac{1}{2} - x_3\right)\lambda r_y$ $\boxed{?}$</td>
<td>$V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1 - 2x_3)\lambda r_y$ $\boxed{?}$</td>
<td></td>
</tr>
<tr>
<td>content provider Y $\frac{1}{2}\lambda r_y$ unchanged</td>
<td>$x_3\lambda r_y &lt; \frac{1}{2}\lambda r_y$ $\boxed{+}$</td>
<td>$x_3\lambda r_y &lt; \frac{1}{2}\lambda r_y$ $\boxed{+}$</td>
<td></td>
</tr>
<tr>
<td>content provider G $\frac{1}{2}\lambda r_G$ unchanged</td>
<td>$\frac{1}{2}\lambda r_G$ unchanged</td>
<td>$\frac{1}{2}\lambda (r_G - (1 - 2x_3)r_y) &lt; \frac{1}{2}\lambda r_G$ $\boxed{+}$</td>
<td></td>
</tr>
<tr>
<td>consumer surplus $\frac{t}{4}$ unchanged</td>
<td>$t\left(x_3^2 - x_3 + \frac{1}{2}\right) &gt; \frac{t}{4}$ $\boxed{-}$</td>
<td>$\frac{t}{4}$ unchanged</td>
<td></td>
</tr>
<tr>
<td>social welfare $V(\lambda) - \frac{t}{4} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_y + \frac{1}{2}\lambda r_G$ $\boxed{+}$</td>
<td>$V(\lambda) - t\left(x_3^2 - 2x_3 + \frac{1}{2}\right) - \frac{d}{\mu - \lambda} + x_3\lambda r_y + (1 - x_3)\lambda r_G$ $\boxed{?}$</td>
<td>$V(\lambda) - \frac{t}{4} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_y + \frac{1}{2}\lambda r_G$ $\boxed{+}$</td>
<td></td>
</tr>
</tbody>
</table>
**Appendix A: List of notations**

\( N \): total number of end consumers (to be normalized to 1)
\( \lambda \): Poisson arrival rate of content requested from each consumer in packets per unit of time
\( V(\lambda) \): the gross value function of retrieving content for each consumer; concave and twice-differentiable
\( \mu \): capacity of the broadband provider in packets per unit of time
\( F \): the fixed fee per unit of time charged by the broadband provider to the end consumers
\( p \): the unit price for priority data packet transmission in per packet per unit of time
\( d \): customers’ delay cost (i.e., congestion cost) per unit of time
\( r_y \): content provider \( Y \)’s revenue rate per request for content
\( r_G \): content provider \( G \)’s revenue rate per request for content
\( t \): fit cost for an end consumer away from the ideal content
\( x \): the marginal consumer indifferent between content providers \( Y \) and \( G \).

**References:**


Washington D.C. Dec 1, 2005: D05.


Appendix B: Derivations of consumer surplus and social welfare for short-run problem

<table>
<thead>
<tr>
<th></th>
<th>Consumer Surplus</th>
<th>Social Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>NN (benchmark)</td>
<td>(1) $\frac{t}{4}$</td>
<td>(2) $V(\lambda) - \frac{t}{4} - \frac{d}{\mu - \lambda} + \frac{1}{2} \lambda r_y + \frac{1}{2} \lambda r_G$</td>
</tr>
<tr>
<td>NNN (Case A)</td>
<td>(3) $t\left(x_1^2 - x_2 + \frac{1}{2}\right) &gt; \frac{t}{4}$</td>
<td>(4) $V(\lambda) - t\left(x_1^2 - 2x_2 + \frac{1}{2}\right) - \frac{d}{\mu - \lambda} + x_1 \lambda r_y + (1-x_2) \lambda r_G$</td>
</tr>
<tr>
<td>Better off</td>
<td></td>
<td>Increased</td>
</tr>
<tr>
<td>NNN (Case B)</td>
<td>(5) $\frac{t}{4}$</td>
<td>(6) $V(\lambda) - \frac{t}{4} - \frac{d}{\mu - \lambda} + \frac{1}{2} \lambda r_y + \frac{1}{2} \lambda r_G$</td>
</tr>
<tr>
<td>Unchanged</td>
<td></td>
<td>Unchanged</td>
</tr>
</tbody>
</table>

(1) Consumer Surplus$_{NN}$

$$\int_{0}^{\frac{t}{2}}\left(V(\lambda) - tx - \frac{d}{\mu - \lambda} - F_{NN}\right)dx + \int_{\frac{t}{2}}^{1} \left(V(\lambda) - t(1-x) - \frac{d}{\mu - \lambda} - F_{NN}\right)dx$$

$$= \int_{0}^{\frac{t}{2}}\left(V(\lambda) - tx - \frac{d}{\mu - \lambda} - (V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda})\right)dx + \int_{\frac{t}{2}}^{1} \left(V(\lambda) - t(1-x) - \frac{d}{\mu - \lambda} - (V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda})\right)dx$$

since $F_{NN} = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$

$$= \int_{0}^{\frac{t}{2}}\left(\frac{t}{2} - tx\right)dx + \int_{\frac{t}{2}}^{1} \left(tx - \frac{t}{2}\right)dx = \left(\frac{t}{2} x - \frac{t}{2} x^2\right)_{0}^{\frac{t}{2}} + \left(\frac{t}{2} x^2 - \frac{t}{2} x\right)_{\frac{t}{2}}^{1} = \frac{t}{4}$$

We can further divide consumer surplus into two parts:

Y’s consumer surplus$_{NN} = \int_{0}^{\frac{t}{2}}\left(V(\lambda) - tx - \frac{d}{\mu - \lambda} - F_{NN}\right)dx = \left(\frac{t}{2} x - \frac{t}{2} x^2\right)_{0}^{\frac{t}{2}} = \frac{t}{8}$.

G’s consumer surplus$_{NN} = \int_{\frac{t}{2}}^{1} \left(V(\lambda) - t(1-x) - \frac{d}{\mu - \lambda} - F_{NN}\right)dx = \left(\frac{t}{2} x^2 - \frac{t}{2} x\right)_{\frac{t}{2}}^{1} = \frac{t}{8}$.

(2) Social Welfare$_{NN}$

= Profit$_{broadband provider}$ + Payoff$_y$ + Payoff$_G$ + Consumer Surplus

$$= V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \left(\frac{1}{2} \lambda r_y\right) + \left(\frac{1}{2} \lambda r_G\right) + \left(\frac{t}{4}\right)$$

$$= V(\lambda) - \frac{t}{4} - \frac{d}{\mu - \lambda} + \frac{1}{2} \lambda r_y + \frac{1}{2} \lambda r_G$$

(3) Consumer Surplus$_{NN}$ Case A
Since $F_3 = V(\lambda) - tx_3 - \frac{d}{\mu - \lambda} = V(\lambda) - t(1-x_3) - \frac{d}{\mu - (1-x_3)\lambda}$,

$$
= \int_0^{x_3} \left( V(\lambda) - tx - \frac{d}{\mu - \lambda} - F_3 \right) dx + \int_{x_3}^1 \left( V(\lambda) - t(1-x) - \frac{d}{\mu - (1-x)\lambda} - F_3 \right) dx
$$

$$
= \int_0^{x_3} \left( V(\lambda) - tx - \frac{d}{\mu - \lambda} - (V(\lambda) - tx_3 - \frac{d}{\mu - \lambda}) \right) dx + \int_{x_3}^1 \left( V(\lambda) - t(1-x) - \frac{d}{\mu - (1-x)\lambda} - (V(\lambda) - t(1-x_3) - \frac{d}{\mu - (1-x_3)\lambda}) \right) dx
$$

$$
= \int_0^{x_3} (tx_3 - tx) dx + \int_{x_3}^1 (tx - tx_3) dx = \left[ tx_3x - x^2 \right]_0^{x_3} + \left[ \frac{t}{2} x^2 - tx_3x \right]_{x_3}^1
$$

$$
= tx_3^2 - \frac{t}{2} x_3^2 + \frac{t}{2} - tx_3 - \frac{t}{2} x_3^2 + tx_3^2 = t(x_3^2 - x_3 + \frac{1}{2})
$$

$$
= t \left( x_3 - \frac{1}{2} \right)^2 + \frac{1}{4}
$$

Therefore consumers are better off in NNN Case A.

We can further divide consumer surplus into two parts: 
Y’s consumer surplus, NNN Case A

$$
= \int_0^{x_3} \left( V(\lambda) - tx - \frac{d}{\mu - \lambda} - F_3 \right) dx = \left[ tx_3x - \frac{t}{2} x^2 \right]_0^{x_3},
$$

$$
= \frac{t}{2} x_3^2 < \frac{t}{8} \text{ since } 0 < x_3 < \frac{1}{2}
$$

which is less than Y’s consumers surplus under the case of NN.

G’s consumer surplus, NNN Case A

$$
= \int_{x_3}^1 \left( V(\lambda) - t(1-x) - \frac{d}{\mu - (1-x)\lambda} - F_3 \right) dx = \left[ \frac{t}{2} x^2 - tx_3x \right]_{x_3}^1
$$

$$
= t \left( \frac{1}{2} x_3^2 - x_3 + \frac{1}{2} \right) > \frac{t}{8} \text{ (G’s consumer surplus, NNN) since } 0 < x_3 < \frac{1}{2}
$$

(4) Social Welfare, NNN Case A

= Profit, broadband provider + Payoff$_x$ + Payoff$_G$ + Consumer Surplus

$$
= V(\lambda) - tx_3 - \frac{d}{\mu - \lambda} + \left( \frac{1}{2} x_3 \right) \lambda r_G + \left( x_3 \lambda r_y \right) + \left( \frac{1}{2} \lambda r_G \right) + \left( t(x_3^2 - x_3 + \frac{1}{2}) \right)
$$

$$
= V(\lambda) + t \left( x_3^2 - 2x_3 + \frac{1}{2} \right) - \frac{d}{\mu - \lambda} + x_3 \lambda r_y + (1-x_3) \lambda r_G
$$
Social Welfare_{NNN Case A} - Social Welfare_{NN} \\
= \left( V(\lambda) + t \left( x_2^1 - 2x_3 + \frac{1}{2} \right) - \frac{d}{\mu - \lambda} + x_3 \lambda r_\gamma + (1 - x_3) \lambda r_G \right) - \left( V(\lambda) - t \frac{d}{\mu - \lambda} + \frac{1}{2} \lambda r_\gamma + \frac{1}{2} \lambda r_G \right) \\
= t \left( x_2^2 - 2x_3 + \frac{3}{4} \right) - \left( \frac{1}{2} - x_3 \right) \lambda r_\gamma + \left( \frac{1}{2} - x_3 \right) \lambda r_G \\
= t \left( x_2^2 - 2x_3 + \frac{3}{4} \right) + \left( \frac{1}{2} - x_3 \right) \lambda (r_G - r_\gamma) \\
= t \cdot \left[ \left( x_2^2 - \frac{1}{2} \right)^2 + \left( \frac{1}{2} - x_3 \right) \right] + \lambda \left( \frac{1}{2} - x_3 \right) (r_G - r_\gamma) > 0 \\
\text{since } 0 < x_3 < \frac{1}{2} \text{ and } r_G > r_\gamma \\
\text{Therefore, } Social Welfare_{NNN Case A} > Social Welfare_{NN}, \text{ and social welfare increases.} \\

(5) Consumer Surplus_{NNN Case B} \\
= \int_0^{1/2} \left( V(\lambda) - tx - \frac{d}{\mu - \lambda} - F_4 \right) dx + \int_{1/2}^1 \left( V(\lambda) - t(1-x) - \frac{d}{\mu - \lambda} - F_4 \right) dx \\
= \int_0^{1/2} \left( V(\lambda) - tx - \frac{d}{\mu - \lambda} - (V(\lambda) - t (\frac{1}{2} - \frac{d}{\mu - \lambda}) \right) dx + \int_{1/2}^1 \left( V(\lambda) - t(1-x) - \frac{d}{\mu - \lambda} - (V(\lambda) - t \frac{1}{2} - \frac{d}{\mu - \lambda}) \right) dx \\
\text{since } F_4 = V(\lambda) - t \frac{1}{2} - \frac{d}{\mu - \lambda} \\
= \left( \frac{1}{2} - tx \right) dx + \int_{1/2}^1 \left( tx - \frac{t}{2} \right) dx = \left( \frac{1}{2} x - \frac{t}{2} x^2 \right) \Bigg|_0^{1/2} + \left( \frac{t x^2}{2} - \frac{t}{2} x \right) \Bigg|_{1/2}^1 \\
= \frac{t}{4} \\
\text{Therefore consumer surplus remains unchanged.} \\

We can further divide consumer surplus into two parts: \\

Y’s consumer surplus_{NNN Case B}: \\
= \int_0^{1/2} \left( V(\lambda) - tx - \frac{d}{\mu - \lambda} - F_4 \right) dx \\
= \left( \frac{1}{2} x - \frac{t}{2} x^2 \right) \Bigg|_0^{1/2} = \frac{t}{8} = Y’s \text{ consumer surplus}_{NN} \\

G’s consumer surplus_{NNN Case B}: 

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\[
\int_{\frac{1}{2}}^{1} \left[ V(\lambda) - t(1-x) - \frac{d}{\mu - \lambda} - F_4 \right] dx \\
= \left( \frac{t}{2} x^2 - \frac{t}{2} x \right) \bigg|_{\frac{1}{2}}^{1} = \frac{t}{8} = G's \text{ consumer surplus}_{NN}
\]

(6) Social Welfare_{NN} Case B

= Profit_{broadband provider} + Payoff_{Y} + Payoff_{G} + Consumer Surplus

\[
= \left( V(\lambda) - \frac{t}{2} \frac{d}{\mu - \lambda} + (1 - 2x_3) \lambda r_y \right) + \left( x_3 \lambda r_y \right) + \left( \frac{1}{2} \lambda \left( r_y - (1 - 2x_3) r_y \right) \right) \left( \frac{t}{4} \right)
\]

\[
= V(\lambda) - \frac{t}{4} \frac{d}{\mu - \lambda} + \frac{1}{2} \lambda r_y + \frac{1}{2} \lambda r_G
\]

Therefore social welfare remains unchanged.
Appendix C: Empirical data on ISP capacity construction costs

We compare three different types of broadband infrastructure technologies, as shown in the following table (10GBASE-SR is another standard which allows 10Gbps of data rate; however, it is new technology and is not widely used yet). Note that while the different standards correspond to different data rates, the same type of fiber optic cable can be used for different standards. But the different standards require different types of transceivers. Thus, the capacity cost can be thought to have a fixed component (the cable) and one variable component (the transceivers).

<table>
<thead>
<tr>
<th>Standard</th>
<th>Data Rate</th>
<th>Transceivers</th>
</tr>
</thead>
<tbody>
<tr>
<td>10Base-FL</td>
<td>10Mbps</td>
<td>Transition Networks (E-FRL-MC05)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Transceiver</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10 Mbps, Cable connectivity, Package Qty: 1, For: PC systems</td>
</tr>
<tr>
<td>100Base-FX</td>
<td>100Mbps</td>
<td>Transition Networks (SSETF1011-205)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Transceiver</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100 Mbps, Cable connectivity, Package Qty: 1</td>
</tr>
<tr>
<td>1000Base-SX</td>
<td>1Gbps</td>
<td>Transition Networks (SFMFP1314-220)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(SFMFP1314220) Transceiver</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000 Mbps, Cabling: Ethernet 100Base-LX, Ethernet 100Base-SX Package Qty: 1</td>
</tr>
</tbody>
</table>

This suggests the following variable costs as a function of capacity:

<table>
<thead>
<tr>
<th>Suggested Cost Function:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
</tr>
<tr>
<td>μ</td>
</tr>
<tr>
<td>10μ</td>
</tr>
<tr>
<td>100μ</td>
</tr>
</tbody>
</table>

Notes:

10Base-FL: 10BASE-FL is the most commonly used 10BASE-F specification of Ethernet over optical fiber. It replaces the original FOIRL specification, but retains compatibility with FOIRL-based equipment.
Maximum segment length is 2000 meters. Mixing it with FOIRL equipment, maximum segment length reduces to FOIRL’s 1000 meters.

100BASE-FX: 100BASE-FX is a version of Fast Ethernet over optical fiber. It uses two strands of multi-mode optical fiber for receive and transmit. Maximum length is 400 meters for half-duplex connections (to ensure collisions are detected) or 2 kilometers for full-duplex. 100BASE-FX uses the same 4B5B encoding and NRZI line code that 100BASE-TX does. 100BASE-FX is not compatible with 10BASE-FL. It uses a 1.3 micrometer near infrared (NIR) light wavelength.

1000BASE-SX: 1000BASE-SX is a fiber optic Gigabit Ethernet standard. It operates over multi-mode fiber using a 850 nanometer, near infrared (NIR) light wavelength. The specification allows for a maximum distance between endpoints of 220 m over 62.5/125 µm fiber although in practice, with good quality fiber and terminations, 1000BASE-SX will usually work over significantly longer distances. Modern 50/125 µm fibers can reliably extend the signal to 500 m or more. This standard is highly popular for intra-building links in large office buildings, co-location facilities and carrier neutral internet exchanges.

10GBASE-SR: 10GBASE-SR ("short range") is one standard for 10 Gigabit Ethernet. 10 Gigabit Ethernet or 10GbE is the most recent (as of 2006) and fastest of the Ethernet standards. It defines a version of Ethernet with a nominal data rate of 10 Gbit/s, ten times faster than gigabit Ethernet. 10 gigabit Ethernet is very new, and it remains to be seen which of the standards will gain commercial acceptance. 10GBASE-SR is designed to support short distances over deployed multi-mode fiber cabling, it has a range of between 26 m and 82 m depending on cable type. It also supports 300 m operation over new, 50um 2000 MHz·km multi-mode fiber (using 850 nm).

Appendix D: Proof of $\frac{\partial x_3}{\partial \mu} > 0$

\[
t x_3 + \frac{d}{\mu - \lambda} = t(1 - x_3) + \frac{d}{\mu - (1 - x_3) \lambda}
\]

\[
t(2x_3 - 1) + \frac{d}{\mu - \lambda} - \frac{d}{\mu - (1 - x_3) \lambda} = 0
\]

Taking the partial derivative on both sides, we get

\[
2t \frac{\partial x_3}{\partial \mu} + \frac{-d}{(\mu - \lambda)^2} - \frac{d}{(\mu - (1 - x_3) \lambda)^2} = 0
\]

\[
\left(2t + \frac{d \lambda}{(\mu - (1 - x_3) \lambda)^2}\right) \frac{\partial x_3}{\partial \mu} = \frac{d}{(\mu - \lambda)^2} - \frac{d}{(\mu - (1 - x_3) \lambda)^2}
\]

Since RHS > 0 and $2t + \frac{d \lambda}{(\mu - (1 - x_3) \lambda)^2} > 0$, $\frac{\partial x_3}{\partial \mu} > 0$. 